LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2023**

UMT 1502 – CALCULUS

Date: 09-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A		
	Answer ALL the Questions		
1.	**Answer the following (5 x 1 = 5)**		
a)	State Leibnitz theorem.	K1	CO1
b)	Define curvature of a curve at a point on the curve.	K1	CO1
c)	Evaluate $\int \tan^2 x \, dx$.	K1	CO1
d)	Define evolute of a curve.	K1	CO1
e)	Define Beta function.	K1	CO1
2.	**Choose the correct answer (5 x 1 = 5)**		
a)	n^{th} derivative of $\frac{1}{ax+b}$ is a) $\frac{(-1)^{n+1}(n+1)!a^{n+1}}{(ax+b)^{n+1}}$ b) $\frac{(-1)^n \, n! \, a^n}{(ax+b)^n}$ c) $\frac{(-1)^n \, n! \, a^n}{(ax+b)^{n+1}}$ d) $\frac{(-1)^n \, n! \, a^{n+1}}{(ax+b)^{n+1}}$	K1	CO1
b)	Parametric formula for radius of curvature is a) $\frac{(x'^2 - y'^2)^{3/2}}{x'y'' - y'x''}$ b) $\frac{(x'^2 + y'^2)^{3/2}}{x'y'' + y'x''}$ c) $\frac{(x'^2 + y'^2)^{3/2}}{x'y'' - y'x''}$ d) $\frac{(x'^2 - y'^2)^{3/2}}{x'y'' + y'x''}$	K1	CO1
c)	If f is an even function, then $\int_{-a}^{a} f(x)dx$ is _________ a) $2\int_{-a}^{a} f(x)dx$ b) $2\int_{0}^{a} f(x)dx$ c) $2\int_{a}^{0} f(x)dx$ d) 0	K1	CO1
d)	Reduction formula for $\int_0^{\pi/2} sin^n x \, dx$ is a) $I_n = \frac{(n+1)}{n} I_{n+1}$ b) $I_n = \frac{(n-1)}{n} I_{n-1}$ c) $I_n = \frac{(n-1)}{n} I_{n-2}$ d) $I_n = \frac{(n-2)}{n} I_{n-2}$	K1	CO1
e)	$\Gamma(n)$ is (a) Convergent (b) Divergent (c) Oscillating (d) None of these	K1	CO1
3.	**Fill in the blanks (5 x 1 = 5)**		
a)	If $f'(a) = 0$ and $f''(a) \neq 0$, then $f(x)$ has a _________ if $f''(a) < 0$.	K2	CO1

b)	The equation of the tangent at P(x_1, y_1) is _______.	K2	CO1
c)	$\int_0^1 \int_0^1 (x+y)\,dx\,dy =$ _________________.	K2	CO1
d)	$\int \sqrt{a^2+x^2}dx =$ ………………	K2	CO1
e)	$\int_0^\infty e^{-x^2}\,dx =$…………………	K2	CO1
4.	**State True or False** **(5 x 1 = 5)**		
a)	The method of Lagranges multipliers is a technique applied to determine the local maxima and minima of a function of the form f(x,y,z) subject to equality constraints.	K2	CO1
b)	The subtangent to the curve $y = a^x$ is $2x$.	K2	CO1
c)	The tangent to the evolute at any point on it is the normal to the curve at the corresponding point on the curve.	K2	CO1
d)	$\int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx.$	K2	CO1
e)	$\Gamma(n+1) = (n+1)\,\Gamma(n-1).$	K2	CO1

SECTION B

Answer any TWO of the following. **(2 x 10 = 20)**

5.	Show that the maximum value of $x^2y^2z^2$ subject to the restriction $x^2+y^2+z^2 = a^2$ is $\left(\frac{a^2}{3}\right)^3$.	K3	CO2
6.	If $y = \sin(m\,sin^{-1}x)$, prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$ and show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$	K3	CO2
7.	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx.$	K3	CO2
8.	Evaluate $\int_0^1 x^m \log(\frac{1}{x})^n$dx	K3	CO2

SECTION C

Answer any TWO of the following. **(2 x 10 = 20)**

9.	Derive an angle between the radius vector and the tangent and hence find the angle at which the radius vector cuts the curve $\frac{l}{r} = 1 + e\cos\theta$.	K4	CO3
10.	(i) Evaluate $\int_0^{\frac{\pi}{2}} \frac{(sinx)^{\frac{3}{2}}}{(sinx)^{\frac{3}{2}}+(cosx)^{\frac{3}{2}}} dx$ (ii) Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1).	K4	CO3
11.	By changing order of integration evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}$dx dy.	K4	CO3
12.	Prove that $\int_0^{\frac{\pi}{4}} \log(1 + tan\theta)\,d\theta = \frac{\pi}{8}\log 2.$	K4	CO3

SECTION D

Answer any ONE of the following. **(1 x 20 = 20)**

13.	Find the value of the integral $\int\int\int xyz\ dxdydz$ taken through the positive octant of the sphere $x^2+y^2+z^2 = a^2$.	K5	CO4
14.	Prove the relation between Beta and Gamma functions.	K5	CO4

SECTION E			
Answer any ONE of the following.			**(1 x 20 = 20)**
15.	(i) Establish a reduction formula for $\int sin^n x\ dx$, where n is a positive integer and hence find $\int_0^{\frac{\pi}{2}} sin^n x\ dx$. (15 marks) (ii) Show that in the equiangular spiral r = a $e^{\theta cot\alpha}$, the tangent is inclined at a constant angle to the radius vector. (5 marks)	K6	CO5
16.	(i) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ (15 marks) (ii) A rectangular box without a lid is to be made from $12m^3$ of cardboard. Find maximum volume of such a box. (5 marks)	K6	CO5

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